MATH 323: Calculus III

MATERIALS

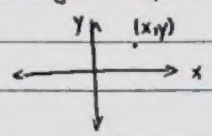
- WebAssign (will include textbook) - around \$120
- Gradescope (for assignment submission) - free
- Website (includes syllabus, calendar, practice problems

- SECTION 12.1: Coordinates in 3-space -

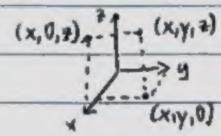
IDEA of Calculus III: Extend Calculus I, II to functions of several variables

Some Geometry in 3-Space

$\mathbb{R}^2$



$\mathbb{R}^3$



shadow of (x,y,z) in $\mathbb{R}^2$ where z is set to 0

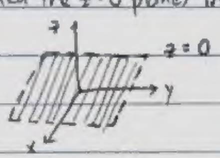
I. Coordinate Planes

- A coordinate plane is a set of points in which a specified coordinate is 0.

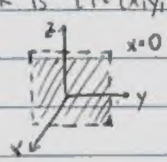
Ex #1: The xy-plane (aka the $z=0$ plane) in $\mathbb{R}^3$ is $\Pi = \{P=(x,y,z) \in \mathbb{R}^3 : z=0\}$

$\in$ = "in"

$:$ = "such that"

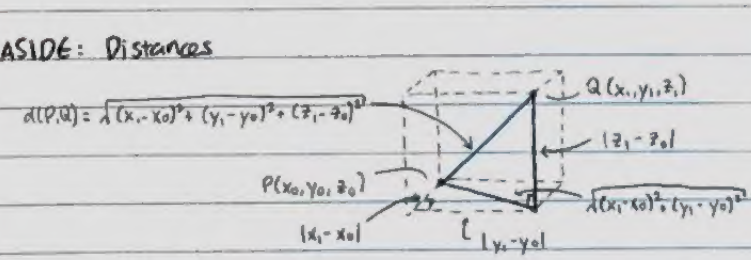


Ex #2: The yz-plane in $\mathbb{R}^3$ is $\{P=(x,y,z) \in \mathbb{R}^3 : x=0\}$



* Try to turn multivariable problems into single variable ones.

ASIDE: Distances



THM (Distance Formula): For $P=(x_0,y_0,z_0)$ and $Q=(x_1,y_1,z_1)$ in 3-space, the distance between P and Q is $d(P,Q)=\sqrt{(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2}$.

II. Spheres

- Let $r > 0$ and let $P \in \mathbb{R}^3$. The sphere of radius r centered at P is $S = \{ Q \in \mathbb{R}^3 : d(P,Q) = r \}$

  If P has coordinates $P = (x_0, y_0, z_0)$, then
  $$S = \{ Q \in \mathbb{R}^3 : d(P,Q) = r \}$$
  $$= \{ (x_1, y_1, z_1) \in \mathbb{R}^3 : \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2} = r \}$$
  $$= \{ (x_1, y_1, z_1) \in \mathbb{R}^3 : (x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2 = r^2 \}$$

Spheres are "surfaces of a hollow ball" (NOT SOLID)

A solid ball is defined by $(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2 \leq r^2$.

NB: "nota bene" or "note well"

* Everything we have done so far has analogs in higher dimensions as well.

Ex: $\mathbb{R}^4$ (4-space) $= \{ (x, y, z, w) : x, y, z, w \in \mathbb{R} \}$ has distance formula.

$$d((x_0, y_0, z_0, w_0), (x_1, y_1, z_1, w_1)) = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2 + (w_1 - w_0)^2}$$

– SECTION 12.2: Vectors –

- A vector in $\mathbb{R}^3$ is a directed line segment, where two vectors are equivalent when they are linear shifts.